Auxiliary telescopes absolute laser tilt determination: the Rayleigh case

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ABSTRACT

We present here a new technique aimed to measure the Laser Guide Star (LGS) upward tilt. As well known, this information together with the LGS tilt measured using the main telescope allow to retrieve, neglecting focus anisokinetism, the Natural Guide Star (NGS) overall tilt. The described technique is similar in its practical implementation to the two auxiliary telescopes methods introduced sometimes ago. However it reduces the most important practical limitations of this methods: namely the use of two different telescopes each one freely moving around the main observatory on distances of the order of several 100m. In its most complete version the techniques uses signal from sodium and Rayleigh LGSs coupled together to avoid geometrical effects that reduce the accuracy of the determinated downward tilt. However a simpler form of the technique uses only the Rayleigh return signal. This configuration has disvantages on the tilt measurement accuracy but result in an easier practical implementation. We report the mathematical description of the tip-tilt retrieval process used in this technique together with numerical result quantifying the tip-tilt determination process accuracy. These results show the achievable performances and existing limitations when using this techniques for tip-tilt estimation from LGSs in the case of large astronomical telescopes.

Keywords: laser guide star, focus anisoplanatism, auxiliary telescopes

1. INTRODUCTION

In the framework of the current efforts to solve the tip–tilt indetermination problem of Laser Guide Stars (LGSs hereafter), various perspective–based schemes has been proposed[1–3]

Such perspective–based schemes rely on some Natural Guide Stars (NGS hereafter) well beyond the isoplanatic patch of the scientific target under observation. Some gauging is performed on the differential tilt between the LGS and such NGS and the absolute tilt can be worked out. These schemes, like the others proposed to solve the same problem[4–6] must face with both fundamental limitations and practical implementation problems.

We deal in the following with the Two Auxiliary Telescope Technique (TATT) described in ref.(1). This technique is affected by the practical problem to require a large area[7] around the main observatory where one have to displace the auxiliary telescopes. This area is characterized by a linear dimension l roughly given by:

$$l \approx \phi^* \times H_{\mathrm{beacon}} \tag{1}$$

where ϕ^* is the typical angular distance required to find out a NGS able to produce an useful tip–tilt signal and H_{beacon} is the altitude of the LGS. It is straightforward to point out that the adoption of a Rayleigh beacon instead of a mesospheric Sodium one will translate into a much smaller l. Typical values for l range in the several hundreds of meters when $H_{\mathrm{beacon}} \approx 95km$ as for the Sodium layer. It is evident that a drastic reduction of the covered area is obtained using some $H_{\mathrm{beacon}} \approx 20km$. Being the former proportional to l^2 the reduction of the ground area to be covered by the auxiliary devices is of the order of $\approx 7\%$ of the original one.

However the conical anisokinetism effect *amplified* by the lower altitude of the beacon acts to reduce the achievable accuracy using this tilt retrieving scheme. In addition this Rayleigh variation of the auxiliary telescopes perspective technique is more sensitive to $C_n^2(h)$ distribution rather than the original mesospheric implementation.

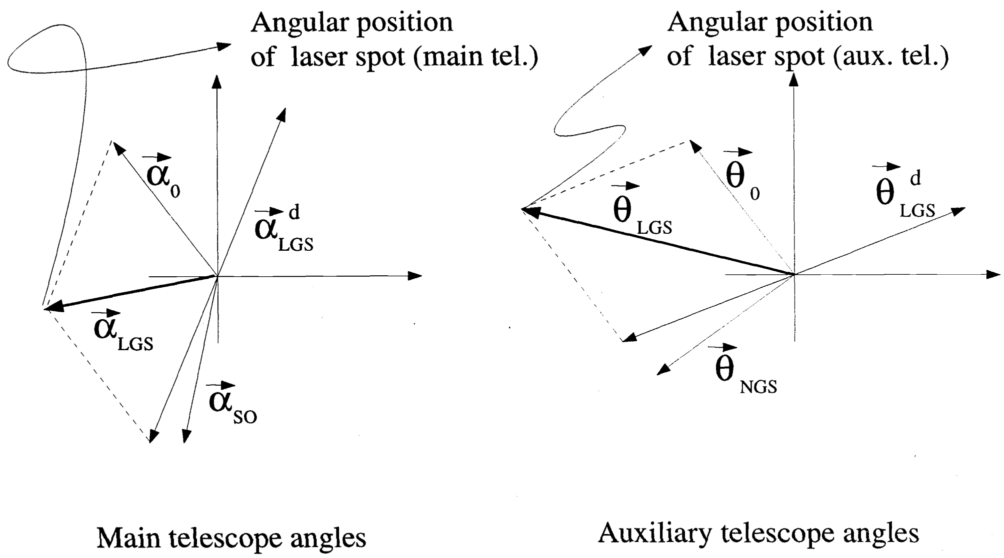

Figure 1. Representations of angles involved in TATT on main and auxiliary telescopes.

In this paper the calculations carried out to preliminary assess the performances attainable lowering the laser guide star heigh are shown, together with a pair of examples concerning two different C_n^2 distribution. One is an Hufnagel–Valley (HV27) profile[8] the other is a modified Hufnagel–Valley as introduce by Beckers.[9] Both atmospheres are normalized to give $r_0 = 0.2$m at 0.55 μ m.

2. MEASUREMENT ERROR OF AUXILIARY TELESCOPE TECHNIQUE

The TATT, as explained in literature,[1] is based upon a differential tilt measurement obtained using an auxiliary telescope at a certain distance from the main telescope. This measure allows to evaluate the laser guide star (LGS) spot position on the scatterer layer. This information together with a measurement of the LGS as seen from the main telescope is enough to single out the downward LGS tilt. In this process the differential measurement is performed using the LGS and a natural field star that can be well outside the isokinetic patch of the main telescope. Referring to fig.1 we can write the following equations in which we consider a tilt angle positive or negative respectively for upward and downward propagation.

$$\alpha_{LGS} = \alpha_0 - \alpha^d_{LGS} \tag{2}$$

$$\theta_{NGS} = -\theta^d_{NGS} \tag{3}$$

$$\theta_{LGS} = \theta_0 - \theta^d_{LGS} \tag{4}$$

where $\alpha_0, \alpha^d_{LGS}, \alpha_{LGS}$ are the off-axis, the downward and the overall tilt of the LGS as seen from the main telescope and $\theta_0, \theta^d_{LGS}, \theta_{LGS}$ are the off-axis, the downward and the overall tilt of the LGS as seen from the auxiliary telescope. Moreover θ_{NGS} is the tilt of the field star as seen from the auxiliary telescope. It is easy to observe that α_0 and θ_0, the two angles due to the off-axis propagation of the laser beacon are equal. Rearranging the three equation stated before we obtain

$$\alpha^d_{LGS} = \alpha_0 - \alpha_{LGS} = (\theta_{LGS} - \theta_{NGS}) - \alpha_{LGS} - \left(\theta^d_{NGS} - \theta^d_{LGS}\right) \tag{5}$$

Now, as seen from fig.1 the tilt of the scientific object is different from the LGS downward tilt measured on the main telescope. Using the identity

$$\alpha^d_{SO} = \left(\alpha^d_{SO} - \alpha^d_{LGS}\right) + \alpha^d_{LGS} \tag{6}$$

we obtain the equation for α^d_{SO} as estimated with the TATT to be

$$\alpha^d_{SO} = (\theta_{LGS} - \theta_{NGS}) - \alpha_{LGS} - \left(\theta^d_{NGS} - \theta^d_{LGS}\right) + \left(\alpha^d_{SO} - \alpha^d_{LGS}\right) \tag{7}$$

The last two term of the right member of the equation represent the errors introduced in the estimated tilt of the scientific object. This two term are not null because of the focus anisokinetism effect[10,11] arising on the main and the auxiliary telescope respectively. Considering the two errors as uncorrelated we can obtain the angular tilt error variance that result

$$\sigma^2_{SO} = \sigma^2_\alpha + \sigma^2_\theta \tag{8}$$

3. STATISTICAL EVALUATION OF MEASUREMENT ERROR

The focus anisokinetism error variance has been estimated from different authors.[10–12] Following calculations developed in ref.(10) we can express the focus anisokinetism phase variance for a sodium guide star as

$$\sigma^2_{fa} = 0.59 \cdot (D/d_0)^{5/3} \tag{9}$$

where D is the telescope diameter and d_0 is the LGS parameter introduced by Fried.[13] Moreover we have the general expression for σ^2_{fa} that results[12]

$$\sigma^2_{fa} = cost. \int_0^H dz C^2_n(z) w(z/H) \tag{10}$$

where the weighting function $w(z/H)$, for the tilt mode, result proportional to $(z/H)^2$. This allow us to rewrite eq. 9 , in the following form

$$\sigma^2_{fa} = 0.59 \cdot (D/d_0)^{5/3} \cdot (H_s/H_{LGS})^2 \tag{11}$$

Using eq. 8 for the error variance together with the last equation we can express the final phase error variance on the scientific object tilt determination that results

$$\sigma^2_{SO} = 0.59 \cdot (D_{main}/d_0)^{5/3} \cdot \left[1 + (D_{main}/D_{aux})^{1/3}\right] \tag{12}$$

Here a factor, $(D_{main}/D_{aux})^2$ has been introduced to rescale the phase tilt variance between the auxiliary and the main telescope.

4. NUMERICAL RESULTS

We discuss here result for an 8m class telescope operating in the visible at $0.55\mu m$. Fig. 2 show the behavior of phase error variance versus laser spot height. This has been calculated from two C^2_n profile. The first is an Hufnagel-Valley profile,[8] the second is a modified Hufnagel-Valley profile[9] that has a surface layer accounting for the 70 % of the overall turbulence. Both profiles are rescaled to give $r_0 = 0.2m$ at $0.55\mu m$. However cause of the different height distribution of the refraction index fluctuation, regarding focus anisokinetism, they can be considered as a *bad seeing case* and a *good seeing case.*

The values of the error phase variance showed in fig. 2 do not allow to retrieve diffraction limited strehl ratios. However the plot shows how the reference star heigh can be lowered still reaching useful Full With Half Maximum (FWHM) of the corrected Point Spread Function (PSF). As an example lets consider a reference star height of 30 km that corresponding to a ground area reduction of about 90 %. We found in this case that depending on the C^2_n profile the FWHM ranges between 0.08 and 0.15 *arcsec*. This demonstrate how this new approach to the TATT obtain interesting improvements of the FWHM reducing drastically the ground area required to have a full sky-coverage.

It is interesting to point out that, using an appropriate temporal gating of the laser beacon, non elongated strip could be obtained. This would permit the use of a single auxiliary telescope to perform the two dimensional tilt measurement. Finally it is clear that the effect limiting the usefulness of this technique is the focus anisokinetism and his dependence with height. If this effect can be solved, at least on the main telescope, using multiple laser guide star[14] then the technique could be improved further than shown here.

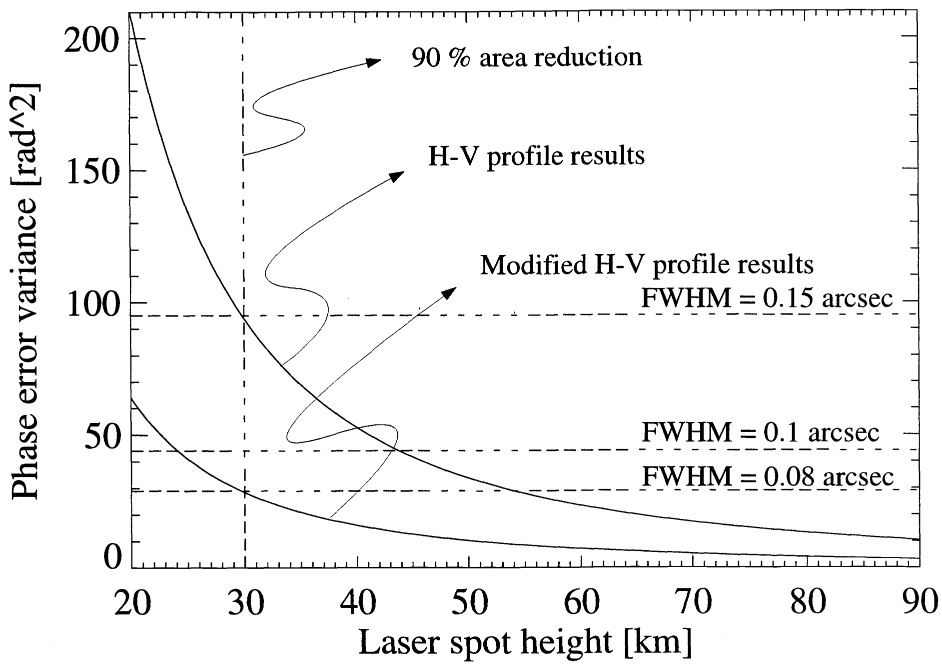

Figure 2. Focus anisokinetism versus source height

5. SODIUM PLUS RAYLEIGH TILT RETRIEVAL SCHEME

In order to overcome conical anisoplanatism for the high–order correction there is no doubt that a Mesospheric Sodium LGS is to be used. In this framework it is attractive from the technical and practical point of view to use the same Sodium Laser Projector both for the Rayleigh portion (that can be used for the tip–tilt retrieval) and for the higher order compensation by resonant backscattering in the Sodium layer.

However it is easy to point out that the Sodium LGS can be used for a further improvement of the tip–tilt retrieval technique described in this paper.

It is almost clear that the auxiliary telescope technique applied at the Rayleigh beacon is equivalent to estimate the *true* angular position of such an artificial reference spot on the sky. Provided that this position is known there is still an error due to focal anisokinetism and effective layer height with respect to the value that could be obtained by a coaxial NGS. We write this error with $\sigma_{\rm Ra}^{\rm FA}$. In an analog manner we can introduce the quantity $\sigma_{\rm Na}^{\rm FA}$ that could be obtained at the expenses of a much larger ground occupation of the auxiliary telescopes movement. It can be shown that[10] :

$$\frac{\sigma_{\rm Na}^{\rm FA}}{\sigma_{\rm Ra}^{\rm FA}} \approx \frac{H_{\rm Ra}}{H_{\rm Na}} \tag{13}$$

Let say γ the correlation between the tilt of the Rayleigh and Sodium beacon of the same Laser projector, as seen from the main telescope. It is expected that this correlation could be significantly close to the unit although not exactly because of the different atmosphere sampled, the different weights of the turbulent layers in the effective layer height, and so on.

Using the auxiliary telescope technique to retrieve the absolute tilt of the Rayleigh beacon and adding it the differential tilt between the two beacons will leads to an overall error $\sigma_{\rm Ra+Na}$ that, under certain assumption, is given by:

$$\sigma_{\rm Ra+Na}^2 \approx \left(\sigma_{\rm Na}^{\rm FA}\right)^2 + (1-\gamma)\left(\sigma_{\rm Ra}^{\rm FA}\right)^2 = \left(\sigma_{\rm Ra}^{\rm FA}\right)^2 \left[\left(\frac{H_{\rm Ra}}{H_{\rm Na}}\right)^2 + 1 - \gamma\right] \tag{14}$$

From the last relationship one can turn out the condition on γ to have the modification of the technique to be effective (that is to be characterized by an error lower than the one attainable by the only–Rayleigh approach). The threeshold occours for $\gamma > 0.07$ for a $H_{\mathrm{Ra}} = 25\mathrm{Km}$.

A detailed evaluation of the coefficent γ and the estimation of the performances attainable with this modification of the technique is beyond the limit of this paper and deserves further calculations.

6. CONCLUSION

We have described a variation of the two auxiliary telescopes technique that allow to reduce its practical implementation. We have discussed the focus anisokinetism error behavior with source heigh. Calculations show that interesting results in terms of tilt phase error variance can still be obtained lowering the laser guide star spot height. Moreover we argument that using the information of both sodium spot plus the low altitude spot allow further reduction of the error , previously evaluated, in the case of a single spot. Finally we point out that if focus anisokinetism effect can be ruled out the proposed technique can couple diffraction limited performances on both Strehl ratios and FWHM.

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