

Radio plasma fringes as guide stars: tracking the global tilt

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ABSTRACT

We present a novel technique to alleviate the problem of the global tilt in artificial guide stars for adaptive optics. This technique is based on the registration of trails of radio-excited plasma spots caused by the atmospheric tilt. Following the time trace of the trails one can find and measure the tilt produced by atmospheric turbulent layers. Different methods were applied to estimate the extent of the trails. We describe results of computer simulations, showing the performance of the proposed approach.

Key words: adaptive optics, plasma beacons, radio guide stars, atmospheric plasma, artificially ionised regions, radio astronomy, radio interferometry

I. INTRODUCTION

In the past several years a number of decameter class ground-based telescopes have been built and more are now under construction. All of them are still limited by the distorting effects of the Earth's atmosphere, which limits their effective coherent aperture, and are thus provided with adaptive optics. Adaptive optics allows to measure and undo the effects of turbulence in real time, delivering near-diffraction-limited performance at the infra-red and visible wavelengths. Many include or plan on adding artificial guide stars, which serve as reference beacons for precise measurement of the turbulence even when nearby stars are too weak for this purpose.

Unfortunately, artificial guide stars suffer from the problem of determination of the ascending beam global tilt, which is difficult to measure and remove from the descending beam. Because of reciprocity of propagation paths, a conventional synthetic beacon is unable to sense a full aperture tilt and can only be used to measure the higher-order distortions. For this reason, the problem of indeterminable tilt of artificial guide stars is recognised as a fundamental problem to be overcome before applying a full adaptive optics correction for any type of seeing condition. The current solution is to use a further natural star just for measurement of the tilt, complicating the optics and processing of the adaptive optics.

Since laser guide stars (LGSs) play a key role in the achievement of full sky diffraction capabilities at visible wavelengths, the efforts of the world astronomical community in the past years have concentrated on the development of the LGS technology. However, this solution has many problems, both practical and conceptual, slowing its application. For this reason, one of authors has proposed an alternative approach to the creation of artificial reference source. This method is based on the use of visible plasma spots excited by fringes between intense radio waves^{1,2}. Three distant radio dishes are phased together to have 1 m fringes tens of kilometers above the telescope, the fringes spanning up to tens of meters. The air breaks down where the radio intensity is the highest, and creates plasma, visible in oxygen and nitrogen lines. Despite the attraction of the mentioned approach, the problem of lack of a useful tip-tilt signal is not solved in the case of radio guide stars (RGSs), just as it is not solved in the case of the LGSs. Therefore, the solution of the global tilt problem is very important for RGSs too.

The global tip-tilt problem has been attacked in several ways and now one can count more than ten techniques to sense it⁵. A couple of these techniques were tested experimentally on the sky. Namely, the statistical technique devised by Belen'kii⁶ was tested on Polaris using a Rayleigh laser beacon⁷. Shortly afterwards, a perspective based technique⁸ was tested by comparison on a couple of stars using a sodium laser guide star⁹. Both experiments are open loop ones, i.e. no attempt has been made for real-time correction of tip-tilt, but data have been collected and later reduced in order to establish the merits

of the technique. Although both experiments show a remarkable good agreement with the expected results, it is worth pointing out that tilt correlation does not exceed correction of more than seventy to eighty percent. This means that more work needs to be done in order to provide a reliable system able to produce high Strehl ratios. The Kolmogorov turbulence power spectrum is peaked at the very low frequencies, making the tip-tilt component very important. Hence only very accurate tip-tilt removal can prevent significant deterioration of the achievable Strehl ratio¹⁰.

Even RGSs cannot solve the problem in full, since the pointing accuracy of the radio beams is limited due to their large wave length. In addition, atmospheric effects are estimated⁴ to swing the interfering beams at about 0.1 m at the elevation of 50 km. In addition, local winds up to 40 m/s might carry the plasma spot away⁴, although its elevation can be chosen to be minimally affected by the wind. To make things easier, since the pulse repetition rate of the radio beams is chosen to leave residual plasma for the next pulse (to save on power⁴), this repetition rate can be tuned to have a known spot drift.

We suggest a new concept for absolute tilt retrieval based on RGS technology. Because of the large scale of the radio fringes, this solution has direct implications also for the fields of radio astronomy, radio interferometry and optical interferometry, laser communications and power beaming (see below). In all these fields, there is a need for a local reference frame to help phase the elements of the radio dish, or co-phase the different dishes to serve as one coherent unit. The added capability to track tilts as proposed here will enhance these devices.

III. THE TECHNIQUE

Tilt, produced by atmospheric turbulent layers, can result in a trail of the spot of the artificial guide star visible from the telescope. In considering this effect, we pose a question: is it a positive or a negative effect and can it be used to break down the reciprocity and to sense a full aperture tilt with an artificial guide star?

To reply this question, let us suppose that the upward interfering beams exciting the beacon are characterised by an upward wandering $\underline{b}(t)$. In what follows, we only consider the central fringe; inclusion of more fringes (which move in unison) can only improve the method discussed below. Let us also suppose that the excited spot has a certain decay time t_0 . In other words, if one excites the beacon with a single short pulse of intensity I_0 , one will observe a beacon intensity that changes with time accordingly to:

$$I = I_0 \exp(-t/t_0)$$

For the mesospheric sodium layer, excited by laser light at the same D1 and D2 lines¹¹, t_0 is of the order of pSec. The effect of the trails caused by turbulent atmosphere tilt is negligible and hence useless. In exciting the oxygen or nitrogen in a plasma cloud using radio waves, the decay time t_0 can take much longer values. Because the recombination of plasma varies according to the density of the atmosphere, the decay time will depend upon the height of the excitation. At the altitude of 60 km the decay time of plasma is of the order of 0.01 sec, comparable with the time required for the excited beam to be registered. The repetition rate is chosen so as to leave exactly this residual plasma for the next pulse. Hence, we will see simultaneously the recently excited spot and some residue of previous spots next to it. As a time-series, this looks like a bright spot with a trail behind it. This is similar in appearance to an over-exposed image intensifier showing trails behind bright moving spots. Therefore, in the case of RGS the trails of the excited plasma spot, produced by tilt, will be visible from the ground-based telescope.

Since the elevation of the plasma can be tuned, t_0 will vary with it, for instance, to be of the same order of the Greenwood frequency. However, there is no need to have t_0 locked to some specific frequency related to the atmospheric parameters.

Except for an ionospheric beacon^{1,2}, the spot and its trail are always contained within an isoplanatic patch. In this situation, we will see the evolution of the beacon. Because of the non-zero decay time t_0 , we will see a sort of evanescent trace that follows the beacon movements. The direction and extension of portion of the trail closer to the beacon is an indication of the variation of the upward tilt. A slowly moving upward tilt will produce a short trail, a fast moving upward tilt will produce a longer trail. For instance, looking at the brightest point in the trail (the current beacon), of brightness I , and at an adjacent point of brightness I/e , one will see exactly an angular separation of

$$s(t) = \underline{b}(t) - \underline{b}(t-t_0)$$

This is, roughly speaking, the derivative of $\underline{b}(t)$. More precisely, this is a finite difference of $\underline{b}(t)$ with respect to t_0 (Figure 1). In any case, by integration or numerical summation one should be able to retrieve $\underline{b}(t)$. Each measurement of the finite difference will be affected by some error and the integration error will grow with time leading, at a certain time, to

some unacceptable value. If this happens after long time (compared to t_0), one can lock onto a much fainter natural guide star and close the low frequency loop there. Such an approach was discussed in detail previously³.

Using the assumption that $\underline{b}(t)$ is always within an isoplanatic patch, the measurement of the trail extent and position angle is totally unaffected by the downward tilt, because both $\underline{b}(t)$ and $\underline{b}(t-t_0)$ are observed at the same time and are in the same isoplanatic patch. Hence, this downward tilt has no effect on the differential evaluation of the derivative of the upward tilt.

Therefore, the full aperture tilt of the atmosphere can be found and measured following the time trace of the trails of the radio beacon. It means that the proposed technique permits the sensing of the full-aperture tilt of the atmospheric wave front distortions using a RGS, eliminating one of the fundamental limitations of adaptive optics.

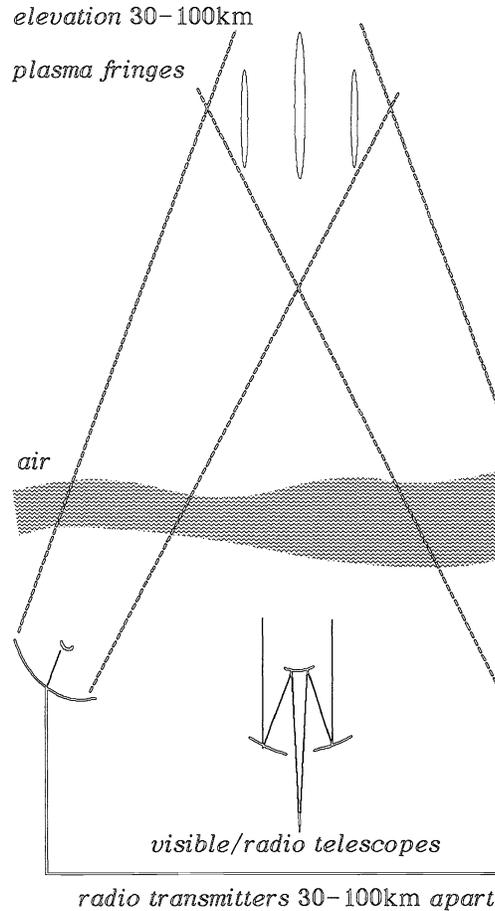


Figure 1: Radio guide stars are created as radio beams interfere and the air breaks down at the crests of the fringes.

III. SIMULATIONS

To check out the proposed approach we ran many computer simulations. Specifically, we generated a series of RGS spots using the Kolmogorov spectrum for the upward tilt, common to the three radio beams involved. The drift in the tilt (and hence the trail) of the radio beams can be as small as 0.1 m, in which case it might be so stable that it does not require any improvement⁴. Even that small a drift can be detected by extending it intentionally, by adding an oscillatory term to the upward beams. Since the radio beams are interfered to create bright fringes, all one has to do is change the phase of the beams relative to each other. Notice, however, that the shift of the spot cannot be too large, since each radio pulse re-

vitalises the plasma from the previous pulse, this being more power efficient than a continuous excitation⁴. The size of the plasma spots depends on the chosen wave length, elevation, and distance between transmitters, and can range between 1 m and tens of meters^{1,2,4}. We used a gaussian profile, of standard deviation of 3-4 m.

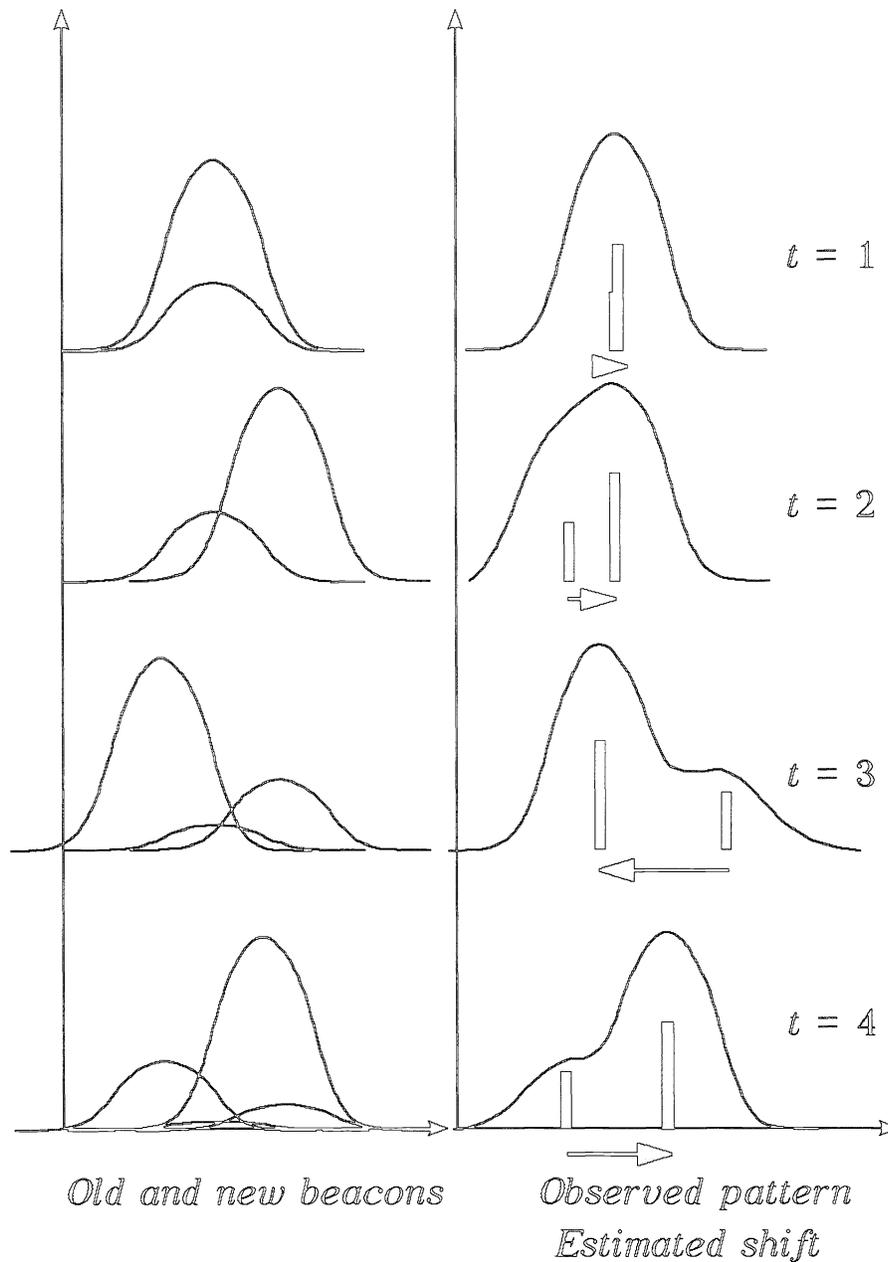


Figure 2: Decay and motion of the plasma spot. On the left are shown the recent few spots, and on the right their (noiseless) appearance. The two bars and arrows are estimates from this observed pattern, where the global tilt is not known.

In the simulation, we created a single plasma spot; for three transmitters one would get at least three, around the telescope, but we only wish to prove the method here. The next shot will create a new nearby spot, while the older one is reduced by a decay factor, chosen between 0.1 and 0.7 (i.e. with decay times different from t_0). The next shot will add yet another new spot, with the two older ones decayed again by the same factor. Some snapshots of these simulations are shown in Figure 2. In analysis, we added Poisson noise to the detected signal: we assumed between 20 and 2000 photons at the

brightest pixel, where each pixel is equivalent to about 1 m at the plasma elevation. Other noise sources, such as detector additive noise, or non-homogenous plasma, were neglected. Consequently we now have a sum of two gaussians, one strong and one weak, and even weaker, the residue of older gaussians (Figure 1). We have to identify the gaussians, so as to detect the most recent one and the distance from it to the previous one. This is a difficult task, even without including noise in the signals. We tried a number of ways to analyse the data.

(a) We first identified the brightest point in the pattern. This is the approximate location of the current spot. We subtract a gaussian from the intensity pattern, and locate the previous spot as being the next brightest point again. This is similar to the CLEAN algorithm used to deconvolve radio maps^{12,13}, except that we stop after only two iterations. The resolution is limited to integer pixels.

(b) Next, we tried to use optimisation: we fitted one gaussian near the brightest pixel, and subtracted it as before. The older spot was located by finding the center of gravity of the residual.

(c) The first two methods suffer from bias in the choice of the first spot, a bias introduced by the nearby second spot. To avoid this bias, we fitted two gaussians simultaneously. The optimisation process involved in the fitting is rather slow.

(d) Another way to tell the new and old spots apart is by comparing the first moment of the intensity pattern and the first moment the *square* of the intensity pattern. Because the older spot is weaker, it will show less on the squared image. Let d be the decay ratio, and the first and second intensity moments be i_1 and i_2 . Call for short the estimates for the previous, current, and separation, a , b , and $s = b - a$, respectively. Now we get

$$\begin{aligned} s &= (i_1 - i_2) / [d^2 / (1 + d^2) - d^{1/2} / (1 + d^{1/2})] \\ b &= i_1 - s \cdot d^2 / (1 + d^2) \end{aligned}$$

(e) In an alternative analysis, we found the first and second coordinate moments of the elongated spot. Assuming a gaussian profile for each spot, we found the analytical values of these moments along the two Cartesian axes. Thus we could find the locations of the two recent spots and hence their separation. Including the location of the brightest spot in the pattern broke the central symmetry of these moments. Let v be the variance of the gaussian spot, x_1 and x_2 the first and second moments, and x_3 the location of the brightest spot. The old and new spots are at

$$\begin{aligned} a &= x_1 - \text{sign}(x_3 - x_1) [(-x_1^2 - v + x_2) / d]^{1/2} \\ b &= x_1 (1 + d) - a \cdot d \end{aligned}$$

(f) Since the spots tend to merge, it is possible to sharpen the images digitally. We used a Wiener deconvolution scheme, in which the theoretical shape of the spot was used as the object, and the noise has Poisson statistics^{14,15}. In the Fourier domain the deconvolved image is

$$I_{deconv} = I_{source} S^* / (SS^* + N),$$

where I_{deconv} , I_{source} and S are the Fourier transforms of the deconvolved image, the original image, and the spot profile. N is the mean power spectrum of the noise, and * symbols conjugation. After deconvolution, the brightest and next brightest spots are identified, as in (a). The accuracy is limited again to integer pixels.

After identification of the two last spots in the pattern, we calculated their distance. We integrated this quantity for a large number of pulses. Although we have quite good knowledge of the statistics of the beam wander, we did not include them in the calculation as a prior. This allows to include other sources of tilt, such as mechanical vibrations, as well as intentional scanning of the beams. In the simulation, we used the known gaussian spot shape for fitting it in the different methods. In the experiment, this profile will have to be measured.

In Figure 2 we see the true location of the trail as created by the radio beacon in the sky, and the calculated location from the data reduction using the different schemes. As shown next, the method works best when the trail extent is the longest and the spots are nearly separated, when the decay in time is the least, and when the noise is negligible. Identification of the brightest location is always quite accurate, but that of the previous one is lacking. This is the main source of error. Still, we could see that the numerical integration did follow the actual beam with time even for quite unfavourable conditions.

In Figure 3 the six methods are compared for different decay values. In this case, the gaussian spot had a standard deviation of 3 m, and the jitter a standard deviation of 4 m (3 and 4 pixels in the simulation, respectively). The mean signal at the brightest pixel was twenty photons, and the simulation included 200 steps. If the true and estimated translation vectors (between old and new spot) at step i were t_i and e_i , what is shown is $[(t_i - e_i)^2 / \sigma_i^2]^{1/2}$. The two methods with the least error (b

and d) are also the methods that tend to slightly underestimate the translation vector - the other methods slightly overestimate it. Corresponding results were obtained for different parameters of the beam.

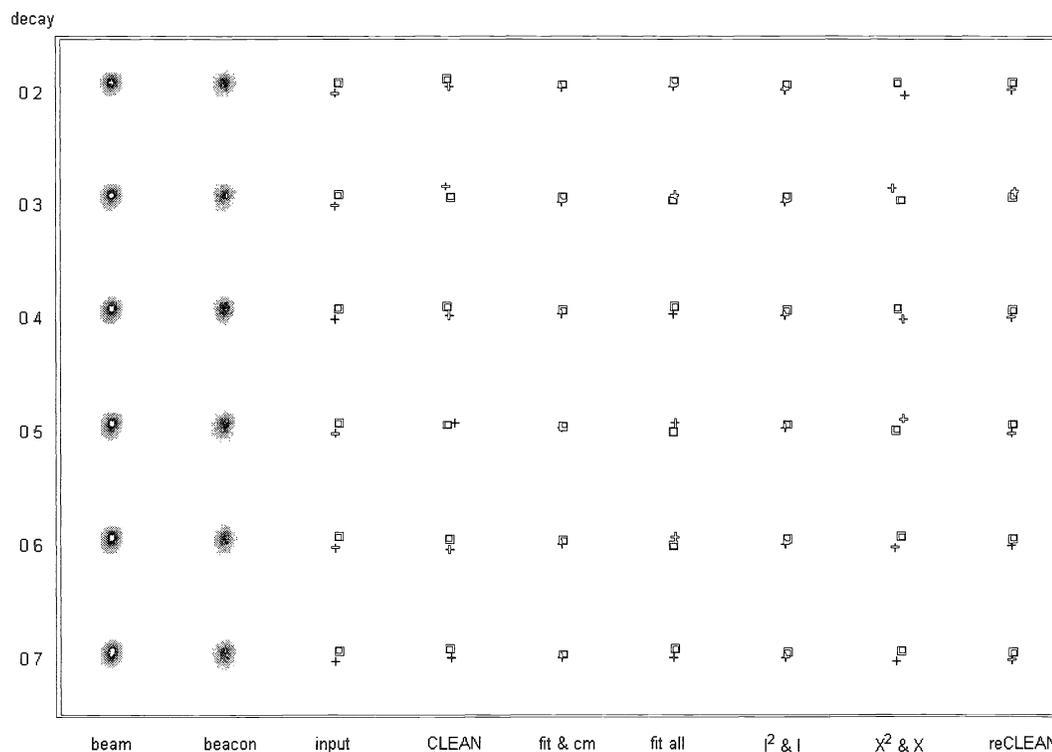


Figure 3: Sample simulation step: the location of the previous (cross) and current (box) beacons are the same but the decays are different (top to bottom). From left: noiseless signal, signal with noise, true locations, and results of the six estimators. The mean number of photons at the brightest pixel was twenty.

IV. PARAMETERS OF THE METHOD

The process of the creation of trails of RGSs may be more complicated than simulated here, since atmospheric wind dispersion and general diffusion can also result in this effect. These processes tend to spread even the single beacon⁴. In particular, the maximum speed of winds at altitude of about 60 km (where inter-pulse time is about 10 ms) and latitudes between 30 degrees north and 30 degrees south (where most telescopes are located) can reach a reasonable value of about 20 m/s. Tuning of the plasma to an altitude of slower wind is possible. However, in this study we have not introduced any influence of wind on the beacon layer for simplicity. We believe that wind, if constant in direction and intensity, will introduce an additional apparent constant value in the derivative estimate. This can be removed easily in the real-time reduction process, because it is a constant offset in the derivative, estimated at each update loop on the faint natural guide star.

How does the distance between the old and new plasma spot affect our results? Of course, the further the points are the easier they are to separate numerically. If this separation is too small, we can enhance it by scanning the relative phases of the interfering beams. However, sweeping of the spots, artificially or by atmospheric phase or by local wind, cannot exceed a fraction of the spot size: beyond this distance, the seed of ionised air from the last pulse will not be enough to

initiate the new breakdown. This is especially important at lower elevations, where recombination is fast and complete. In the simulation we replaced intentional sweeping by random tilts, created by pure atmospheric spectrum of intentionally large extent.

The effect of noise was also investigated. Since there are only rough estimates for the number of photons available^{1,2,4}, we added in the simulation from a few to an infinite number of photons. We found out that down to 20 photons (at the brightest spot) we could still locate the current and previous beacon, but below that the signal deteriorated too fast. In addition, Poisson noise caused over- or under-estimation of the locations and translation (difference) vectors, because this noise was not included in the simple model of the system and the resultant estimator methods.

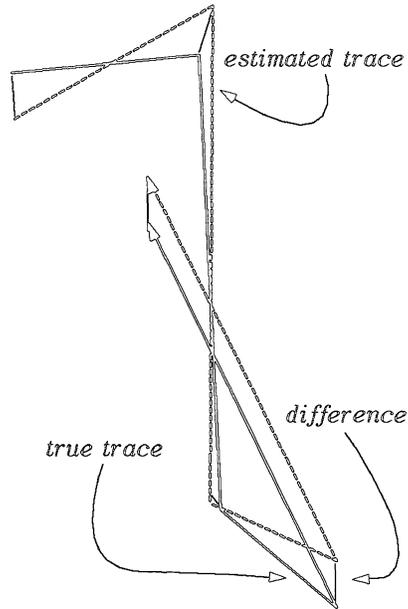


Figure 4: Calculation of the error. At each step the difference between the estimated trace and the true trace is taken. This is the current error, equivalent to subtraction of estimated and true displacement from the previous step.

The effect of decay of the plasma between pulses is also important. Since there are again only rough estimates for this decay, we assumed the intensity of the previous pulse is between 0.1 and 0.7 of the recent one. The stronger the decay, the easier it is to find the location of the two spots. This is partly because in all six methods we assume only two such spots, and neglect the rest of the history. However, when the signal is too weak, a strong decay means low signal-to-noise ratio for the previous spot.

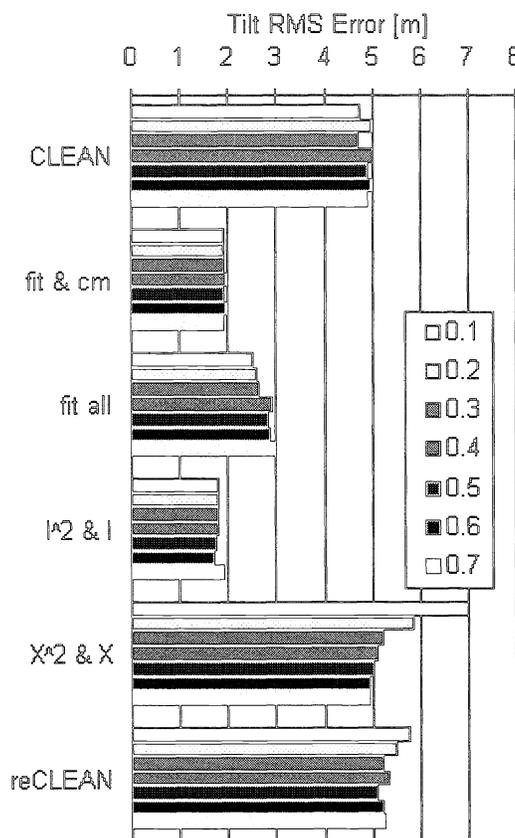
As a result of these simulations, we can say that for each elevation one should be able to find the optimal choice of parameters. At low elevations (30-50 km), the decay is rapid, so the pulses should be close. If the overlap is then too much, constant wind or artificial sweeping are required. Further up more power is needed, the plasma decays more slowly, and the pulses can be spaced apart, but the weaker signal might be a limitation. Above 100 km the plasma pattern is set by other parameters⁴ such as magnetic field lines and is much less controllable.

V. CONCLUSIONS AND APPLICATIONS

We showed that the method of tracking the trails of plasma spots in order to reduce the tilt problem is indeed valid. We tested a number of methods to reduce the data and showed that it is possible to follow the tilting beam with time. The most accurate method was fitting of a gaussian to the plasma spot, and calculation of the center of gravity of the residual. Better results are possible by extension of the fit to even older spots, and by fine-tuning of the chosen parameters for the fit.

We have presented a novel technique that allows solving the global tilt problem in adaptive optics. This technique can be exploited not only in optical astronomy, but also in long base line optical interferometry¹⁶, laser communications and laser power beaming to satellites. Another important application is in radio astronomy and in radio interferometry. In radio

astronomy one needs to phase segmented mirrors and to sense the wave front¹⁷. One of the most severe problems in radio interferometry is the lack of good phase calibration between the dishes^{12,13}. The existence of reference hot spots is a significant step towards solving this problem. As in the visible regime, it should be set outside the central lobe of the radio



dish or interferometer, but still within their field of view.

Figure 5: Comparison of methods at different decay values (0.1 to 0.7 for each method), for signal up to 20 photons/pixel. The standard deviation of the beam jitter was 4 m, and that of the beacon size 3 m. Better models of the physical processes should improve all results.

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