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*Letter to the Editor*

# Absolute tip-tilt determination with laser beacons

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**Abstract.** It is shown that the absolute determination of the tip–tilt can be inferred from two artificial laser beacons in the high atmosphere (e.g. in the Sodium layer) generated from two ground stations located away from the observing place. The technique is limited by the required Field of View for such a telescope. Assuming a retro–fit to existing telescopes, the technique is feasible only for small ($D \approx$ 4m) telescopes in very good seeing conditions ($r_0 = 0.20$m). The applicability to largest telescopes or worst seeing condition is possible in some cases with a dedicated telescope's optics.

**Key words:** adaptive optics – laser stars

## 1. Introduction

Laser Guide Stars (LGSs) can be used as a reference source in astronomical adaptive optics observations (Foy & Labeyrie, 1985). LGSs can be generated both in the 10...20 Km range via Rayleigh scattering or by scattering and/or optical pumping of the 90...120 Km height Sodium layer (Happer *et al.*, 1994). The main fundamental problems of these types of beacon as a reference source in adaptive optics are the focus anisoplanatism (Goad, 1991; Welsh & Gardner, 1991) and the lack of tip–tilt information (Rigaut & Gendron, 1992; Olivier *et al.*, 1993). Focus anisoplanatism is negligible for a $D$ = 4m class telescope with a Sodium LGS and for larger telescopes can be resolved, in principle, via multiple LGSs (Tallon & Foy, 1990) while the problem becomes severe for Rayleigh LGSs, even if several authors (Gonglewski *et al.*, 1992; Christou, Ellerbroek & Fugate, 1993; Ragazzoni & Marchetti, 1995) have pointed out how a Rayleigh LGS is able to offer limited performances, but on a broader isoplanatic patch; in the following it is assumed that Sodium LGSs only are taken into consideration. Lack of tip–tilt information is due to wandering of the laser beam in the propagation to the layer, undistinguishable from the additional wandering of the LGS as observed from the ground. Use of a nearby Natural Guide Star (NGS) inside some isoplanatic patch is a mandatory solution, in this condition. Nevertheless the sky coverage, especially at the shorter wavelength, can be significantly smaller than the unity and, depending upon the diameter $D$ of the telescope, this limitation can be strong for relatively small telescopes; a detailed discussion is given by Rigaut & Gendron (1991) and Olivier *et al.* (1993). Absolute tip–tilt determination still remain a fundamental problem for visible wavelengths and, up to now, the excitation of polychromatic beacons (Foy *et al.*, 1992, 1995) and the use of moving auxiliary telescopes looking at natural stars close to the line of sight to the LGS (Ragazzoni, Esposito & Marchetti, 1995) has been proposed to solve the problem.

## 2. Properties of a transverse laser beacon

Suppose (see Fig.1) to observe from the ground a Sodium laser beacon generated by a ground based station located at a distance $d$ away and illuminating the Sodium layer approximately on the zenith of the observer (this configuration is also called *bistatic* comparing to the *monostatic* where the laser beam is projected coaxially with the telescope's observer, Beckers, 1993).

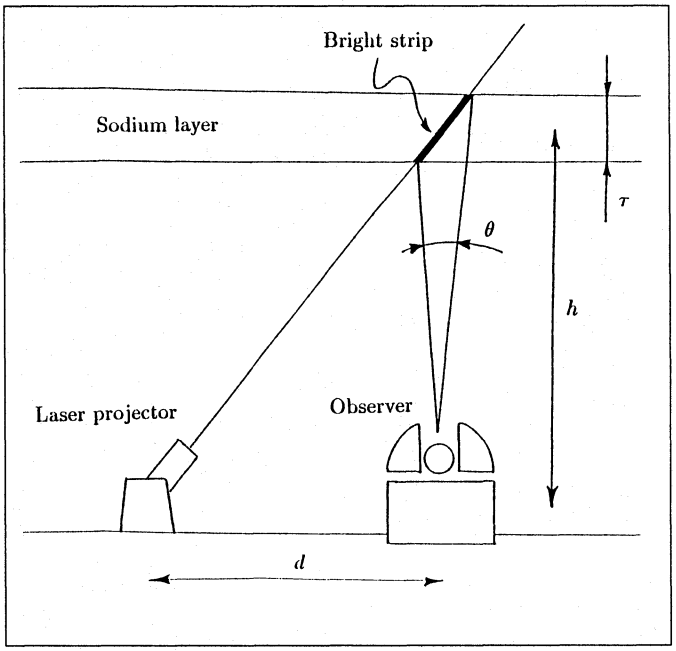

**Fig. 1.** The bistatic configuration allow the observer to see an artificial strip of angular extension $\theta$.

The observer will see a strip of angular length $\theta$ given by:

$$\theta \approx \frac{rd}{h^2} \tag{1}$$

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where $\tau$ is the thickness of the Sodium layer.

Due to the propagation of the beam in the lower, first 20 Kms, region of the atmosphere the exact position of the strip as it is observed from the ground is not known *a priori*. However, some properties of this strip can be pointed out:

- Light propagation effects are negligible because $\tau/c \ll f_G^{-1}$ where $f_G$ is the Greenwood frequency (Greenwood, 1977) and $c$ is the speed of light. Moreover, the amount of turbulence in the Sodium layer region can be neglected. As a consequence the physical strip is straight, taking into considerations only the effect induced by the propagation from the laser to the Sodium layer.
- The apparent position of the strip can translates of an angular amount $\phi$ comparable to the seeing wandering as observed from the ground based laser station through a telescope of the same diameter $D_p$ of the laser beam projector. This angle is a gaussian random variable with an rms $\sigma_\phi$ given by the following relationship:

$$\sigma_\phi \approx 0.4 \left(\frac{\lambda}{r_0}\right)^{5/6} \left(\frac{\lambda}{D_p}\right)^{1/6} \frac{\sqrt{d^2+h^2}}{h^2} \qquad (2)$$

  where here, and in the next eq.(4), we used the value of 0.4 for the numerical constant. In the literature one can derive values ranging from 0.413 (Acton, 1995; Brandt, Mauter & Smartt, 1987), 0.420 (O'Byrne *et al.*, 1995), 0.423 (Sarazin & Roddier, 1990) to 0.427 (Olivier *et al.*, 1993). Because of the spreading of such a *constant* and of our rough estimations we choose to adopt simply 0.4. The angle given by eq.(2) is, in the case of practical interest, much smaller than the isoplanatic patch size. Care is to be given to the fact that the $r_0$ adopted in eq.(2) is the one characterizing the site at a distance $d$ from the observatory. The last term in the same relationships take into account for the longer propagation path for the upwarding beam.
- The strip can be apparently rotated on the sky of the same angle, resulting in a rotation effects at its ends of the order of $\phi \times \theta$ usually absolutely negligible with respect to the wandering of a star as observed from the ground-based telescope, given by eq.(2) but replacing $D_p$ with $D$.

The conclusion is that the strip is randomly translated in a *rigid* manner of a quantity $\sigma_\phi$ (in an rms sense) due to atmospherical perturbation at the laser beam projector site while the strip will be observed as distorted by the telescope's observer due to atmospherical perturbation from the beacon to the telescope.

These departures from a straight line can be measured, at least in principle, in the direction perpendicular to the strip main length $\theta$ (Fig.2) obtaining $N$ independent estimations $\psi$ where $N$ is given by the ratio between the observed length strip $\theta$ and the tilt isoplanatic patchsize, sometimes referred as *isokinetic* patch size. Its value reported in the literature is $\approx 0.3D/\bar{h}$ (Beckers, 1993; O'Byrne, 1995) or $\approx 0.26D/\bar{h}$ (Roddier *et al.*, 1993) and it is roughly confirmed by observations (McClure *et al.*, (1991); Sivaramakrishnan, Weymann & Beletic, 1995); detailed calculations shows a dependence slighlty different upon $D$: from Sandler *et al.* (1994) one can derive, for instance, a proportionality with $D^{6/5}$. Again, assuming for the numerical constant the simplest 0.3 one obtain:

$$N \approx \frac{\theta\bar{h}}{0.3D} \qquad (3)$$

where $\bar{h}$ is an averaged altitude that depends upon the $C_n^2(h)$ distribution and from the author estimate (Shapiro, 1976; Fried, 1979; Chassat. 1989) but it is usually in the range 5...10 Km. The described approach assumes a full decorrelation between the $N$ strips, while most of the estimates of the isokinetich angles refers to some decorrelation value. The correction factor to be applied should be, however, of the order of the unity and it is neglected in the following.

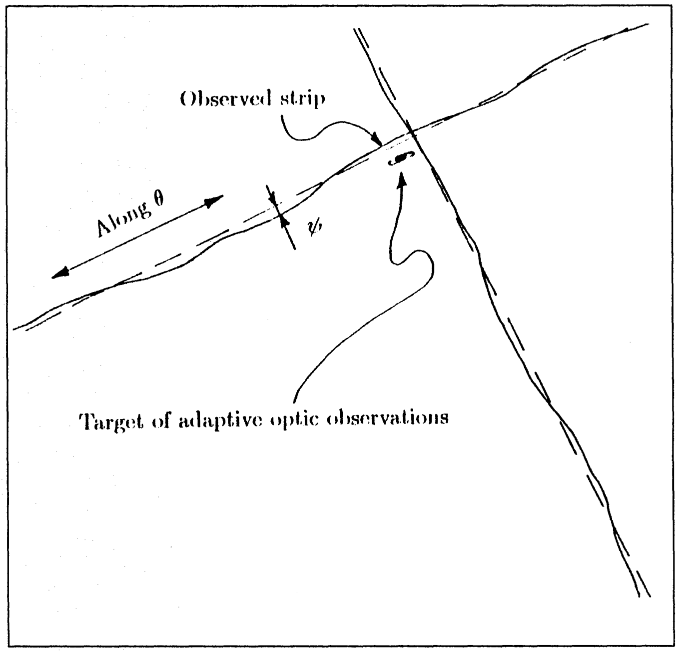

**Fig. 2.** Measuring the distortion of two beacon strips one can derive the absolute bidimensional tilt in a region close to the strips intersection.

Because $<\psi>= 0$, and assuming negligible errors in the relative tilt measurements, one can derive the absolute position of the strip as it should be observed if propagation effects from the Sodium layer to the observer could be removed. From the difference between the straight line fitted to the observed distorted strip and the strip distortion itself, close to the object targetted by the adaptive optics telescope, one can obtain the absolute tilt determination. This can be performed, in the conditions described, with a precision equal to the wandering of stars as seen from the observer over the square root of the number of independent measurements:

$$\sigma_\psi \approx \frac{0.4}{\sqrt{N}} \left(\frac{\lambda}{r_0}\right)^{5/6} \left(\frac{\lambda}{D}\right)^{1/6} \qquad (4)$$

In order to have this uncertainty small enough to be useful one should aim to keep it smaller than the diffraction-limited Airy disk of the telescope's observer. Taking $\sigma_\psi \approx 0.5\lambda/D$ one need:

$$N \approx 0.6 \left(\frac{D}{r_0}\right)^{5/3} \qquad (5)$$

Binding together the equations stated one can establish the required length of the strip $\theta$:

$$\theta \approx \frac{0.2D^{8/3}}{\bar{h}r_0^{5/3}} \qquad (6)$$

and the required distance between the laser projector and the observatory in order to obtain such a strip length:

$$d \approx 0.2 \frac{D^{8/3} h^2}{\tau \bar{h} r_0^{5/3}} \tag{7}$$

Adopting some *nominal* values of $h = 90$Km, $\tau = 10$Km and $\bar{h} = 10$Km various estimations for $N$, $\theta$ and $d$ are given in Tab.1

| | $r_0 = 5$cm | $r_0 = 10$cm | $r_0 = 20$cm |
|---|---|---|---|
| | $N = 891$ | $N = 281$ | $N = 89$ |
| $D = 4$m | $\theta = 6.8^0$ | $\theta = 2.1^0$ | $\theta = 0.66^0$ |
| | $d = 96$Km | $d = 30$Km | $d = 9.5$Km |
| | $N = 2829$ | $N = 890$ | $N = 281$ |
| $D = 8m$ | $\theta = 43.2^0$ | $\theta = 13.6^0$ | $\theta = 4.3^0$ |
| | $d = 611$Km | $d = 190$Km | $d = 60$Km |

**Table 1.** The parameters of the configuration described in the text for various telescopes and seeing conditions.

As it can be seen the proposed configuration is practically unfeasible for $D = 8$m class telescope (because of the large $d$ and, by consequence, side effects from the curvature of the Earth, beam attenuation and tracking requirements at the laser stations) with, perhaps, the exception of very good seeing. For the $D = 4$m class telescope the proposed technique appears feasible assuming that, at least for the LGS imaging, a large Field of View can be covered. An optical system covering a Field of View of some degrees for the single wavelength of the Sodium line used in the LGS, and with a point spread function that along the radial direction can be as bad as the isokinetic patch size is not a formidable task. However it is clear that a retrofit to existing telescopes is rather difficult.

Adopting a limited strip size of $\approx 30$ arcmin (an usual value for the focal plane Field of View of most telescopes) a residual jitter $\sigma_R$ will be obtained. Its value, given in $\lambda/D$ units is given by the following:

$$\sigma_R(\lambda/D) \approx \sigma_\psi(\lambda/D) \times \sqrt{\theta/30'} \tag{8}$$

this latter value is reported in Tab.2 for the cases taken into consideration. These cases are accomplished with $d \approx 4$Km and with some anamorphic relay in the focal plane of the telescope. For the $D = 4$m class telescope reasonable good results for a partial adaptive optics, 100% sky coverage, are still obtained. It is easy to show, in fact, that the gain brought by the described metod is roughly $\theta\bar{h}/0.3D)^{1/2}$ that is of the order of 10 for the cited case.

| $\sigma_R(\lambda/D)$ | $r_0 = 5$cm | $r_0 = 10$cm | $r_0 = 20$cm |
|---|---|---|---|
| $D = 4$m | 1.82 | 1.02 | 0.57 |
| $D = 8m$ | 4.58 | 2.57 | 1.44 |

**Table 2.** The residual rms jitter, due to the adoption of a strip limited to 30': a value that could fit most of the existing telescopes.

In this way, however, only absolute tilt along one axis is obtained and two laser projector at approx right angle positions are needed in order to perform the full absolute tip–tilt detection.

Some words must be spent about the required power on the laser beams. In the following the dependence of light intensity from the scattering angle is neglected because scattering efficiency doesn't change dramatically. In the case of pure Rayleigh scattering the efficiency of un–polarized light is lowered by a factor 2 with a $90^0$ scattering angle while in the described example for $D = 4$m and $r_0 = 10$cm, the scattering angle is somewhat $d/h \approx 20^0$ away from the exact backscattering, where the efficiency reaches a maximum. The adoption of a polarized laser could avoid even this slight effect. A monostatic LGS must provide enough photons on a collection area of the order of $r_0^2$ and the whole photons scattered in the Sodium layer can be used. In this case the whole $D$ of the telescope can be used, but the light is spreaded into $N$ small segments, however because the $N$ measurements are added together, a precision in the measurement $\sqrt{N}$ times worst is still sufficent. Assuming the whole photons of each segment is used for the tip–tilt relative determination the power of this laser should be a factor of the order of $\sqrt{N}(r_0/D)^2$ times the one of the monostatic configuration for full adaptive optics correction. This factor becomes $\approx 0.4 \times (D/r_0)^{-7/6}$ and it is of the order of 0.1 for useful values of $D/r_0$. This means that the power demand on the two laser projectors to be used for the absolute tip–tilt determination is approx one order of magnitude less than the currently developed ones for the LGSs generation. While the cited assumptions are valid also for the tilt determination in the segment within the isoplanatic patch of the observed object (provided that it is much closer to the strip than the isoplanatic patch size, an assumption easily reachable, because the strip is man–driven), a third monostatic LGS is required to perform the full adaptive optics correction. The adoption of the segments close to the observed object in order to estimate the wavefront derivative along two orthogonal direction (one for each laser beam) is possible in principle, but this solution requires $N$ times more power in the two projectors than the one required for the monostatic configuration. It is also to be recalled that the sky brightness, integrated over the whole length of the strip can affects the attainable precision, or increase the power requirements of the laser beams. Another open point regards the homogeneity of the Sodium layer, that is of importance for the proposed method.

Finally it must be pointed out a somewhat increased problem of light pollution in the observatory area due to Rayleigh scattering.

## 3. Conclusions

It has been shown that using two laser projectors located at some distance from the observatory an absolute tip–tilt determination can be obtained for $D = 4$m class telescopes with the today–technology laser beacons. Full adaptive optics correction with 100% sky–coverage requires three lasers, or a system to project the beams from three different places and an *ad hoc* telescope able to image a couple of large strips in the sky. A reduction to a system that could fit into existing telescopes leads to less striking performances, but still with a 100% sky coverage in the visible band. The technique described appears to fail with largest $D$ or worst seeing condition. However it is to be pointed out that a finite outer scale $L_0$ translates into smaller isokinetic patch size for large diameter telescopes (Takato & Yamaguchi, 1995) and from this fact it could follow some effectiveness of the proposed technique even for the

$D = 8$m class telescopes. A detailed error budget of this technique, and a consistent comparison with other techniques for the absolute tip–tilt determination is beyond the limits of this letter and will be the subject of a following paper.

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