Sodium beacon tip–tilt determination with Rayleigh-aided auxiliary telescope technique

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ABSTRACT

The usefulness of tracking the Rayleigh portion of a mesospheric sodium laser guide star as reference for absolute tip–tilt recovery in the frame of the auxiliary telescopes technique is shown. This approach leads to the reduction of the ground occupation needed to attain a given sky coverage by more than one order of magnitude. Speed, tracking precision, and the number of auxiliary telescopes are also reduced, making this new approach a more attractive one. The use of a low-altitude Rayleigh spot reinforces the fundamental limitations affecting this and other techniques, thus degrading significantly the quality of the recovered tip–tilt. However, it is shown that, provided adequate care is taken in the collection and treatment of data, an interesting tilt signal can still be retrieved.

Key words: atmospheric effects – methods: miscellaneous – techniques: miscellaneous – telescopes.

1 INTRODUCTION

Mesospheric sodium laser guide stars, together with adaptive optics instrumentation, could provide diffraction limit capabilities in routine operations for the next-generation large telescopes.

In order to translate this vision into reality, a number of technical and fundamental problems have to be solved on a theoretical and a practical base. Recovery of absolute tip–tilt information (together with conical anisoplanatism) is one of the major limitations on the pathway to the described goal.

Recently a number of works dealt with several techniques aimed at solving, at least in principle, the tilt indetermination problem (Belen'kii 1994, 1995, 1997; Foy et al. 1992, 1995; Ragazzoni, Esposito & Marchetti 1995, hereafter REM95; Ragazzoni 1996a,b, 1997; Ragazzoni & Marchetti 1997).

Efforts to define the fundamental and practical limits to the various techniques have translated into a number of published works (Esposito, Riccardi & Ragazzoni 1996, hereafter ERR96; Neymann 1996). In particular, the REM95 technique has been the subject of a study by our

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group, in order to establish its fundamental limits and the possible ways to overcome, at least partially, the practical problems linked to its implementation. Concerning the latter, the more serious drawback is, without any doubt, the large ground area required around the main telescope. A detailed study by Marchetti & Ragazzoni (1997) shows that REM95, in their original implementation, imply a noticeable realization cost. In this paper we outline a variation of the technique requiring only a small fraction of the effort required to deploy the original REM95 technique. The approach described in the following has a drawback, in that there is a degradation of the recovered tilt: however, it will be shown that, with adequate countermeasures, this can be limited to a reasonable value.

2 MEASUREMENT ERRORS AND GUIDE STAR HEIGHT

A simple approach to reducing the ground working area of the auxiliary telescopes consists of using a lower height artificial star as a target for the auxiliary telescopes (see Fig. 1). However, the downward overall tilt measurement error can depend on the guide star altitude because of two different effects: the focus anisokinetism and the effective layer height. We point out in this paper that the Rayleigh back-scattered portion originating from the laser used for the sodium artifcial star can be used to minimize such errors
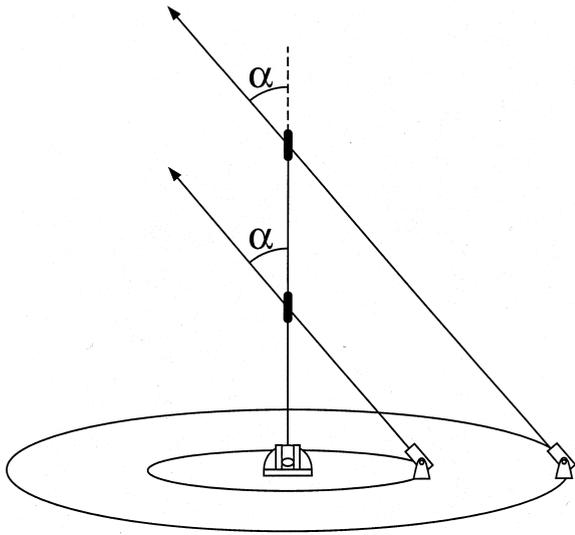
Figure 1. Ground surface occupation is reduced given a lower altitude artifcial source when a maximum off-axis angle α is fixed.

Figure 2. Effect of the finite height of the laser-generated source considering only upward propagation. The knowledge of the h quantity allows us to retrieve the true upward beacon tilt θ_0 using the measured tilt θ_{meas}.

still retrieving the ground coverage advantages. We quantify focus anisokinetism and effective layer height errors in the following sections, with the assumption that the artificial star height is higher than the maximum turbulence height.

2.1 Effective layer height

This error has been considered by Ragazzoni, Marchetti & Brusa (1997, hereafter RMB97). They show how the measured angular tilt, θ_{meas}, of a source at finite distance from the receiving aperture is different from the angular tilt θ_0 introduced by atmospheric wavefront perturbation. This is graphically exemplified in Fig. 2. As described in RMB97, it is possible to define (referring in our case to an upward tilt instead of a downward tilt) an instantaneous effective tilting height h, so that using symbols defined in Fig. 2 results in

$$\theta_0 = \theta_{meas} \frac{H}{H - h}. \tag{1}$$

Experimental measurements allow us to estimate the mean value $\langle h \rangle$ of this tilting height. Following RMB97, the use of $\langle h \rangle$ instead of h in equation (1) leads to an error on the derived quantity θ_0 that turns out to be

$$\sigma_{\theta_2}^2 = \left\langle \theta_0^2 \left(\frac{h - \langle h \rangle}{H - \langle h \rangle} \right)^2 \right\rangle. \tag{2}$$

It is important to note that the variance $\sigma_{\theta_2}^2$ depends on the statistical distribution of θ_0 related to the diameter of the laser projector, D_{proj}. The phase error variance on the main telescope is given by

$$\sigma_{elh}^2 = 2 \left(\frac{\pi D_{main}}{2\lambda} \right)^2 \sigma_{\theta_2}^2, \tag{3}$$

where the factor within the parenthesis is used to convert angular to phase radians. This error can be numerically evaluated following RMB97. However, as stated in REM95,
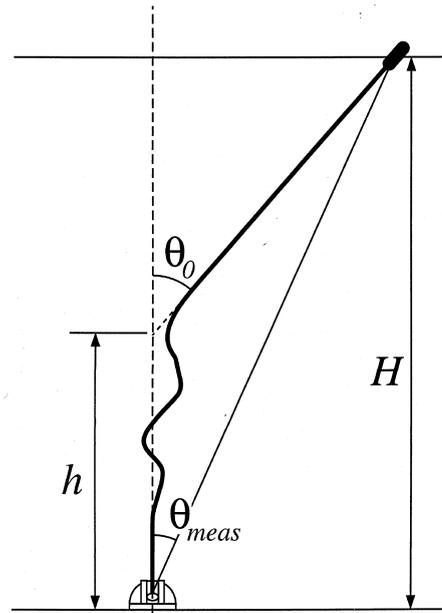
the auxiliary telescope technique is based on a differential tilt measurement between the laser spot and a natural guide star that can be well outside the isokinetic patch of the main telescope. The measurement allows us to estimate the upward laser-spot tilt θ_{meas}. This quantity can be subtracted from the laser-spot tilt as measured on the main telescope to obtain the artificial star downward tilt. Because the upward laser-spot tilts as seen from the main and the auxiliary telescopes are equal, the effect of the instantaneous location of the effective layer height is ruled out.

2.2 Focus anisokinetism error

This error has been considered in detail by ERR96 and by Neymann (1996), and is the result of the difference between the natural guide star tilt and the artificial star tilt, which sample a cylindrical volume and a conical volume of atmosphere, respectively. In the case of the auxiliary telescope technique, using a single reference spot, the two-axis tilt error phase variance σ_{1s}^2 contains only two focus anisokinetism contributions, s_{main}^2 and σ_{aux}^2, the first arising on the main telescope (scientific object and artificial star tilt difference) and the latter on the auxiliary telescope (natural guide star and artificial star tilt difference). Using results obtained in ERR96 regarding the sodium beacon and the results cited in Neymann (1996) about the laser-spot height scaling low, we can evaluate the final phase error variance on the downward wavefront tilt measurements, which result in

$$\sigma_{1s}^2(H) = \sigma_{main}^2(H) + \left(\frac{D_{main}}{D_{aux}} \right)^2 \sigma_{aux}^2(H)$$

$$= 0.6 \left(\frac{D_{main}}{d_{0,Na}} \right)^{5/3} \left[1 + \left(\frac{D_{main}}{D_{aux}} \right)^{1/3} \right] \left(\frac{H_S}{H} \right)^2 \ [\text{rad}^2], \tag{4}$$

where D_{main} and D_{aux} indicate the main and auxiliary telescope diameters respectively, H_S and H are the altitude of a sodium laser spot and of a given laser spot, and $d_{0,Na}$ is the focus anisoplanatism parameter introduced by Fried & Belsher (1994) evaluated in the case of a sodium beacon $(H_S) \approx 90$ km).

We can use equation (4) to numerically evaluate the tilt error variance arising from focus anisokinetism. Throughout our calculations we have assumed, for the main, auxiliary and projector telescopes, a diameter of $D_{main} = 8$ m, $D_{aux} = 0.5$ m and $D_{proj} = 0.5$ m respectively. The results are shown in Fig. 3. They were obtained using two C_n^2 profiles. The first is a Hufnagel–Valley (HV) profile as reported in Sandler et al. (1994), the second is a modified Hufnagel–Valley (MHV) profile introduced by Beckers (1993) that contains a surface layer accounting for 70 per cent of the atmospheric phase variance. These two profiles are normalized so that $r_0 = 0.2$ m at $\lambda = 0.5$ μm and, at this wavelength, they give $d_{0,Na} = 3.0$ m and 6.0 m respectively. They can be considered the bad and good seeing cases from the point of view of focus anisokinetism and effective layer height errors. Thus, values obtained using these two profiles show a good statistical sample of the results that can be achieved using this technique.

3 TWO-SPOT TILT RETRIEVING SCHEME

A possible method by which to retrieve the downward tilt consists of simultaneously generating a sodium and a lower-altitude spot. In this approach the lower spot is targeted by the auxiliary telescope, thus retaining the ground coverage advantages. The additional information on the sodium spot position allows us to further reduce, under favourable turbulence conditions, the tilt error variance of the scientific object calculated in Section 2. We show (below) that a sodium beacon provides enough photons to generate a Rayleigh spot bright enough to perform the measurements needed. This approach allows us to rule out the need for two separate lasers.

3.1 Geometrical configuration

In the two-spot approach we assume a measurement of the overall tilt α_S of the sodium spot as seen from the main telescope. Furthermore, the low-altitude Rayleigh spot position δ_R on the appropriate scattering layer is known from the auxiliary telescope measurement. This information, together with a statistical estimation of the effective layer height position, allows us to evaluate the upward tilt of the sodium spot θ_S. In fact, referring to the symbols and the geometrical configuration shown in Fig. 4, it is easy to see that

$$\theta_S = \frac{\delta_S}{H_S} = \frac{(1 - h/H_S)}{(1 - h/H_R)} \theta_R. \tag{5}$$

The quantity θ_S can be subtracted to the tilt α_S, retrieving the sodium downward tilt. The tilt information obtained from the sodium spot enables us to reduce the focus anisokinetism error contribution to the estimate of the scientific object tilt.
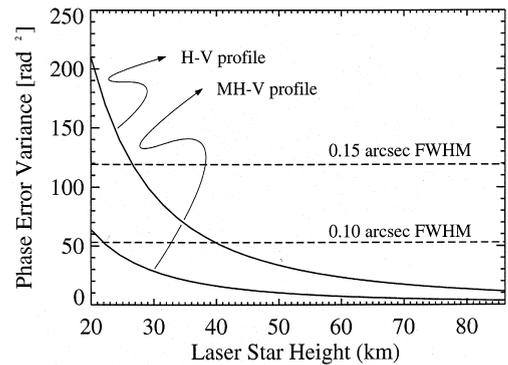
Figure 3. Behaviour of the tilt error variance using a single laser spot. The two curves refer to the HV and MHV profiles. The dashed lines represent the error variances leading to a point spread function FWHM of 0.1 arcsec and 0.15 arcsec. All data are calculated for an 8-m telescope at 0.5 μm.
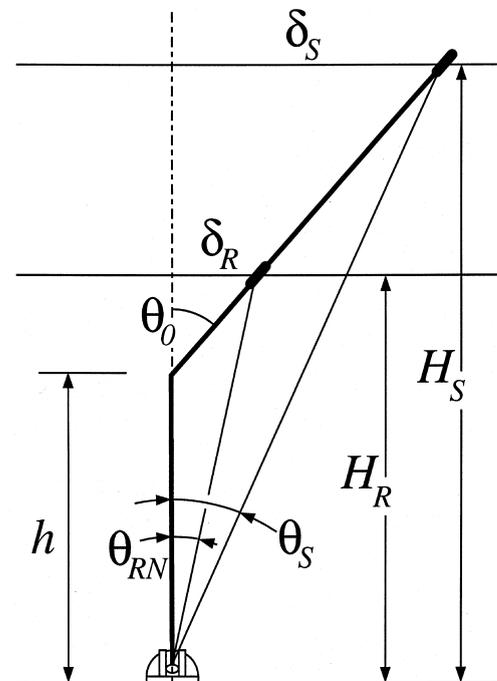

Figure 4. The two-spot configuration arising from the upward propagation of the laser beam is outlined. Rayleigh and sodium spots due to scattered laser photons are located at heights H_R and H_S respectively.

4 TWO-SPOT TECHNIQUE NUMERICAL EXAMPLE

It is shown by Marchetti & Ragazzoni (1997) that, by using the auxiliary telescope technique and one sodium spot, an area of about 1 km² is required to obtain full sky coverage at visible wavelength. If, instead, we take as the target of the auxiliary telescopes a Rayleigh spot located 25 km over the main telescope, the full sky coverage condition is reached using only 10 per cent of the previous area. Moreover, this height reduction of the artificial target linearly reduces the ground speed needed by the auxiliary telescopes to track the laser spot during the exposure time. Both of these effects

greatly simplify the positioning of the auxiliary telescopes. Considering the equations introduced in Sections 2 and 3, we can estimate the accuracy of the scientific object tilt retrieval process when the two-spot technique is used. Throughout the following calculations we assume the same numerical values introduced in Section 2, in particular an 8-main telescope and a 0.5-m auxiliary telescope.

4.1 Rayleigh spot generation

First we evaluate, using the Light Detection and Ranging (LIDAR) equation (Fugate 1996), the number of Rayleigh photons detected by the auxiliary telescope located 50 m away from the main telescope. In this calculation we assume a back-scattering volume centred at an altitude of $H = 25$ km and thickness $\Delta l = 200$ m. Considering the tilt sampling rate at 100 Hz and a pulsed sodium laser of average power P and wavelength λ, we have a number of returned photons N_{ph} per sample given by

$$N_{ph} = 10^{-2} \eta \beta_{BS} \frac{\pi D_{aux}^2/4}{H^2} \Delta l \frac{P}{\lambda/hc}, \tag{6}$$

where η is the product of detector quantum efficiency and telescope optical transmission, β_{BS} is the fraction of incident photons back-scattered toward the receiver per meter of scattering volume, and h and c are the Planck constant and the speed of light respectively. Substituting usual numerical values we obtain an N_{ph} value of about 100 for a 5 W average power laser. This result shows that a sodium beacon can provide enough Rayleigh photons to generate a Rayleigh spot suitable for tilt measurements.

4.2 Scientific object tilt estimation error

The two-spot configuration allows us to evaluate the sodium upward tilt, θ_S. The obtained estimation for θ_S contains two error terms. One depends on the error associated with the measurement of the low-altitude spot position δ_R that is a result of the focus anisokinetism effect on the auxiliary telescope. The other depends on the use, in equation (5), of a statistical estimate of the height h instead of the instantaneous value of the effective layer height. Finally, the error associated with the scientific object tilt is obtained by adding the errors on θ_S to the focus anisokinetism error related to the sodium spot. Considering equation (5) and the formulae introduced earlier for the focus anisokinetism and effective layer height errors, we obtain the following expression for the phase error variance σ_S^2 in the two-spot approach:

$$\sigma_{2s}^2(H) = \left(1 - \frac{H_R}{H_S}\right)^2 \sigma_{elh}^2$$

$$+ \left[\frac{(1 - \langle h \rangle/H_S)}{(1 - \langle h \rangle/H_R)}\right]^2 \left(\frac{D_{main}}{D_{aux}}\right)^2 \sigma_{aux}^2(H) + \sigma_{main}^2(H_S). \tag{7}$$

Fig. 5 reports the results in terms of the tilt error variance of the scientific object. These results are obtained using the same parameters and turbulence profiles introduced in Section 2. The plots show that, depending on the turbulence conditions, the two-spot approach allows us to reduce the
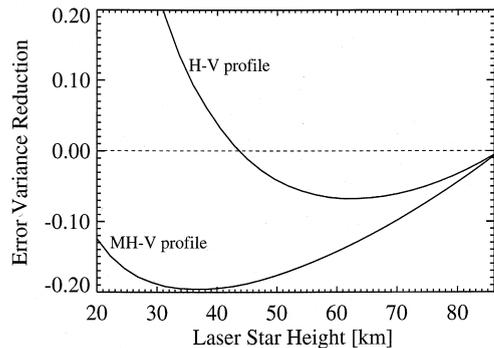

Figure 5. Improvement factor of the error variance obtained using the two-spot approach versus the single-spot one. Negative values mean a reduction of the overall tilt error variance.

overall error variance with respect to the single-spot case. However, as shown in Fig. 2, it must be noted that found variance values do not allow us to retrieve diffraction-limited Strehl ratios. However, the tilt error variance is sufficiently reduced for us to obtain a useful improvement of the point spread function of the full width at half-maximum (FWHM). This is readily shown by the formula expressing the FWHM as a function of the residual tilt phase variance σ_ϕ^2, that is

$$FWHM = \frac{4\sqrt{2 \ln 2}}{\pi} \frac{\lambda}{D_{main}} \sigma_\phi. \tag{8}$$

Let us suppose a Rayleigh spot height of 25 km that corresponds to an area reduction of about 90 per cent. From Fig. 3 and Fig. 5 we found that, in the case of the HV profile, the best results are obtained using the single-spot approach which allows us to reach a FWHM of 0.16 arcsec. However, the two-spot approach gives the best results in the case of MHV profile, allowing us to obtain a 0.08 arcsec FWHM.

Finally we note two things: first, an appropriate grating of the Rayleigh beacon might allow us to obtain a non-elongated Rayleigh spot, useful for two-dimensional tilt measurements, and thus reducing the number of auxiliary telescopes to one. Secondly, let us suppose that the adaptive optic system allows us to correct for focus anisokinetism effects, for example by means of multiple laser guide stars (Ragazzoni, Esposito & Riccardi 1998). In this situation this technique can couple a simplified practical realization to diffraction-limited performances on both FWHM and Strehl ratio values.

5   CONCLUSIONS

We have described some modifications of a proposed technique that greatly simplifies its practical implementation. The effect of these modifications on the measurement error arising from focus anisokinetism and effective layer height is discussed and we show that the tilt error can be reduced considering both Rayleigh and sodium spots. The remaining tilt error is estimated to allow a substantial reduction of the seeing-limited point spread function FWHM, but remains too large to achieve diffraction-limited Strehl ratio values. However, solutions to the focus anisokinetism problem

could enable this technique to couple diffraction-limited performances with a simple, practical implementation.

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